

Welcome to AP Calculus!

Name: _____

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*I likely won't check my email until mid-August, but feel free to reach out and I'll get back to you as soon as I can!

Your first assignment is this packet. If a fellow AP Calculus newbie loses this packet, remind him it can be found on the DHS website. This is a review of the many of the major mathematical concepts you will need to have mastered going into AP Calculus AB and BC. The intent of this packet is to review the skills you have learned in previous courses and to be "warmed up" and ready to go on the first day of class. **Be ready for a test the first day of school!** If you find yourself weak in any areas of this packet, I suggest searching online for resources to help you. Show your work clearly and completely, and use separate paper if you need more space.

Unless otherwise indicated, **NO CALCULATORS** are to be used to complete the following sections (one section indicates the need for a graphing calculator). You may use your calculator to verify answers as needed or suggested by the answer key. **Answers are given at the end of this packet.**

DO NOT START THIS PACKET UNTIL AUGUST! As tempting as it is, it does you no good to do this packet as soon as summer begins. This is the only time I'll ever say this, but procrastinate until the end! Allow yourself a two week window before the first day of school to work on this, and plan on **at least 4 hours to complete this packet...** more if you need to stop and refresh your skills using notes from previous courses or online resources.

I am so excited for a new batch of brilliant math minds I can mold into calculus nerds! See you in the fall!

I) Simplify the following fractions:

$$1) \frac{1}{x} + \frac{1}{y} =$$

$$2) \frac{1}{x} + \frac{1}{x^2} =$$

$$3) \frac{\frac{1}{x} + 1}{\frac{1}{x}} =$$

$$4) \frac{\frac{x}{x+y}}{x} =$$

$$5) \frac{\frac{1}{x+h} + \frac{1}{x}}{x} =$$

II) *Factor each expression:*

6) $x^2 - 16$

7) $x^2 - x - 6$

8) $6x^2 - x - 2$

9) $4x^3 - 19x^2 - 5x$

10) $x^2 + 9$

11) $x^4 - 13x^2 - 30$

12) $x^3 + 27$

13) $x^3 - 8$

14) $(2x-3)^3(x+1) + (x-3)(2x-3)^2$

III) *Solve the following equations/inequalities for x:*

16) $x^2 + 5x - 24 = 0$

17) $x^2 - 9 = 5$

18) $3x^2 - 5x - 2 = 0$

19) $x^2 - 4x = 0$

20) $(x-1)(x^2 - 11x + 30) = 0$

21) $\sqrt{x+1} = 41$

22) $\frac{y}{x+1} = \frac{z}{x}$

23) $\sqrt[3]{x+1} - 4 = -1$

$$24) x^{-2} = \frac{1}{9}$$

$$25) 2\sqrt{x} = x - 3$$

$$26) \frac{8+x}{x} - 5 = 0$$

$$27) x^{-1} = -3$$

$$28) x^{\frac{4}{3}} = 81$$

$$29) 3x^2 - 6x - 24 \leq 0$$

$$30) \frac{2x-1}{(x+2)(x^2+3)} = 0$$

$$31) x^3 - 2x^2 - 5x + 10 = 0$$

IV) *Are the following expressions equal to $\ln 4$?*

$$32) 2 \ln 2$$

$$33) \frac{\ln 8}{\ln 2}$$

$$34) \ln 8 - \ln 2$$

$$35) \ln 4 + \ln 1$$

$$36) \ln 4 \bullet \ln 1$$

$$36) (\ln 2)^2$$

V) *Write an equation of a line based on the given information:*

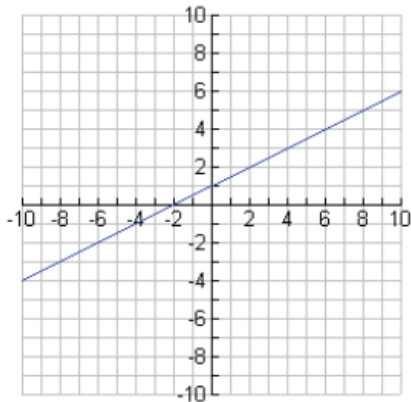
- 37) Find the equation of the line that has a slope of 5 and passes through the point (3, -4).
- 38) Find the equation of the line that passes through the points (4, 1) and (3, -2).
- 39) Find the equation of the line that passes through the points (-2, 1) and is parallel to the line $4x + 2y = -1$.
- 40) Find the equation of the line that has a slope of 0 and passes through the point (-5, 1).
- 41) Find the equation of the line that passes through the origin and is perpendicular to the line $3x + 4y = -7$.
- 42) Find the equation of the line that has an undefined slope and passes through the point (4, -5).

VI) *Intercepts:*

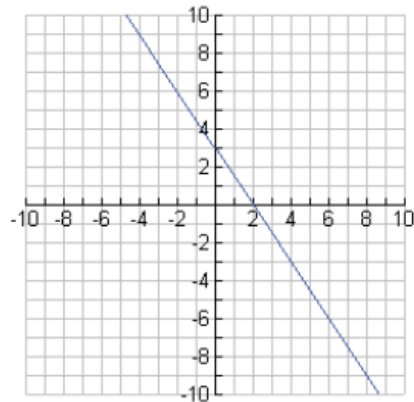
- 43) Find the x and y intercepts of $x^2 + y^2 = 9$.
- 44) Find the equation of the line that has an x -intercept of 5 and a y -intercept of 3.

VII) Write the equation for the following graphs:

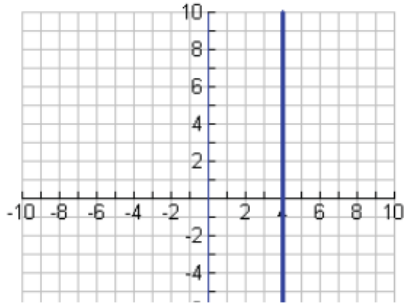
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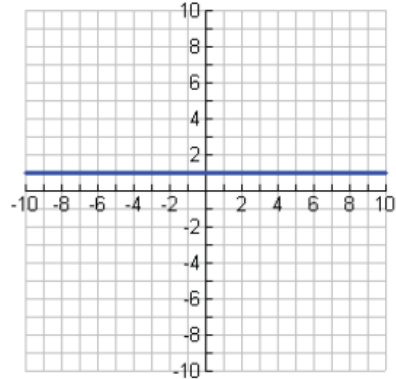
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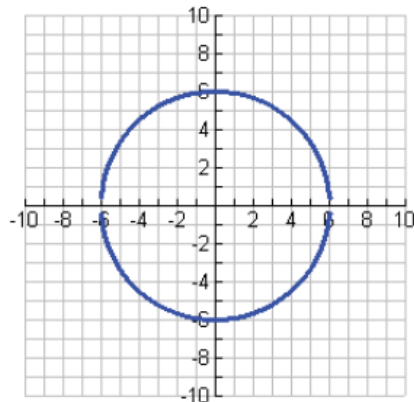
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49)



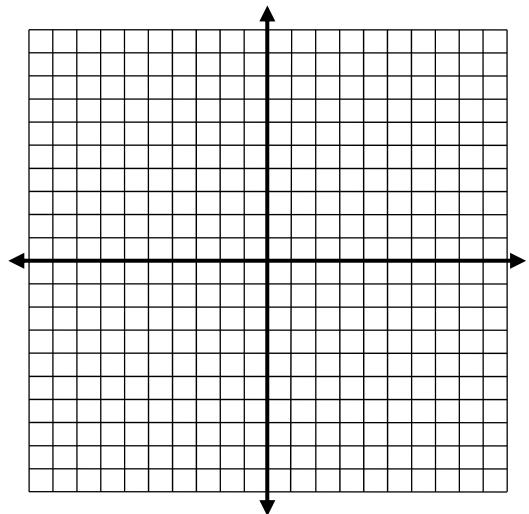
VIII) Given the slope, sketch the following lines:

50) Sketch a line with a slope of 2.

51) Sketch a line with a slope of $\frac{1}{2}$.

52) Sketch a line with a slope of -2.

53) Sketch a line with a slope of $-\frac{1}{2}$.



IX) Sketch the following graphs:

54) $f(x) = 3x + 1$

56) $f(x) = |x|$

58) $x = 3$

60) $f(x) = \ln x$

62) $f(x) = \frac{1}{x}$

64) $f(x) = |x + 1|$

66) $f(x) = x^3 + 1$

68) $f(x) = -x^2 + 1$

70) $f(x) = x^{2/3}$

72) $f(x) = -\frac{1}{x}$

55) $f(x) = x^2$

57) $f(x) = x^3$

59) $y = -4$

61) $f(x) = \sqrt{x}$

63) $f(x) = \frac{1}{x^2}$

65) $f(x) = x^2 + 2x - 3$

67) $f(x) = (x + 1)^2$

69) $f(x) = (x + 1)^{1/3}$

71) $f(x) = e^x$

73) $x^2 + y^2 = 25$

*For IX, sketch all graphs without a calculator and then verify with your calculator. Just draw rough sketches (general shape, intercepts, etc). T-tables may help. For #73, you must solve for y in order to use your calculator. Remember! When you square root both sides of an equation, you need \pm .

X) Rewrite the following functions without absolute values:

74) $f(x) = |x|$

75) $f(x) = |x-1|$

XI) Find the domain and range of each function:

76) $f(x) = x-1$

77) $f(x) = \frac{1}{x}$

78) $f(x) = \frac{1}{x^2+1}$

79) $f(x) = e^x$

80) $f(x) = \sqrt{x-4}$

81) $f(x) = |x-1|+2$

82) $f(x) = \ln x$

83) $f(x) = \sqrt{x^2-3x-4}$

84) $f(x) = \frac{1}{x+6} - 10$

XII) Find the inverse of each function:

85) $f(x) = x+3$

86) $f(x) = \sqrt{x}$

87) $f(x) = \frac{x}{x+2}$

88) $f(x) = \ln x$

XIII) Find the compositions of the function if:

$$f(x) = x^3 + 1, g(x) = x^2 - 2, \text{ and } j(x) = x + 3$$

89) $f(2)$

90) $f(j(x))$

91) $f(j(2))$

92) $g(g(x))$

93) $f(x+h)$

94) $\frac{f(x+h) - f(x)}{h}$

XIV) Solve the simultaneous equations:

95a) $2x + 3y = 8$
 $x + 2y = 5$

95b) $y = x^2 + 2x + 9$
 $7x + y = 19$

96a) The length, l , of a certain rectangle is twice the width, w . Write an equation for the perimeter the rectangle as a function of the width, w .

96b) If the area of the rectangle described above is 50 square feet, find the length and the width of the rectangle.

XV) Intersection of curves:

97) Find the point of intersection between the lines $y = x + 1$ and $3y - x = 5$.

98) Find the point of intersection between the lines $y = x + 7$ and the curve $y = x^2 + 2x + 5$. Also sketch the area between the graphs.

XVI) What do the following mean?

- 99) a graph is in the first quadrant
- 100) $f(2) = 5$
- 101) an expression is a function
- 102) a zero of a function is 4
- 103) y is directly proportional to x (give an example)
- 104) the coefficient of the third term is 5 (give an example)
- 105) a function has only one root
- 106) a function is a polynomial
- 107) two triangles are similar
- 108) a function is even
- 109) a function is odd

XVII) What are the following formulas?

- 110) Quadratic formula
- 111) Pythagorean Theorem
- 112) the hypotenuse of a 45-45-90 isosceles right triangle with a leg of length x .
- 113) the hypotenuse of a 30-60-90 right triangle with shortest leg having a length of x .
- 114) the volume of a sphere
- 115) the volume of a cylinder
- 116) the volume of a cone
- 117) the volume of a box with a square base
- 118) the surface area of a sphere
- 119) the surface area of a cylinder with no top
- 120) the area of a triangle
- 121) the area of a trapezoid
- 122) the cross section through the center of a sphere
- 123) the volume of a prism that has an equilateral triangle with side of length x and height of length y
- 124) area of an equilateral triangle in terms of the length of a side s

XVIII) Solve using similar triangles:

- 125) A six foot man is standing 10 feet away from a 20 foot lamppost. What is the length of his shadow?
- 126) Water is dripping out of a conical figure that has a diameter of 8 inches and a height of 12 inches. When the depth of the water is 8 inches, what is the radius of the water?

XIX) Find the equations of the horizontal and vertical asymptotes of each function:

127) $y = \frac{1}{x-1}$

128) $y = \frac{x^3}{x^3-1}$

XX) Exponent Rules: Which of the following are true?

129) $x^0 = 1$

130) $x^{-2} = \frac{1}{x^2}$

130) $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$

131) $x^5 \cdot x^3 = x^{15}$

132) $x^5 \cdot y^3 = (xy)^5$

133) $(x^3)^5 = x^8$

134) $x^{5-w} = \frac{x^5}{x^w}$

135) $x^{t+5} = (x^t)^5$

136) $\sqrt{\frac{9}{4}} = \frac{3}{2}$

137) $(4x)^{\frac{1}{2}} = 2x$

138) $\sqrt{\frac{1}{x}} = x^{-\frac{1}{2}}$

138) $\sqrt{x^2} = x$

139) $\sqrt{x^2-25} = x-5$

140) $x^{\frac{4}{3}} = \sqrt[4]{x^3}$

141) $\left(x^{\frac{1}{2}} + y^{\frac{1}{2}}\right)^2 = x + y$

142) $x^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{x^2}}$

143) $e^{\ln x^2} = x^2$

143) $\ln e^3 = 3$

144) $e^{2\ln 2 - \ln 5} = \frac{4}{5}$

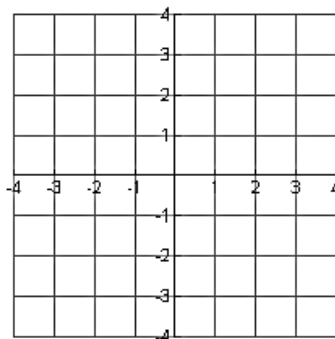
145) $\ln x^2 = (\ln x)^2$

146) Expand using the properties of logarithms: $\ln \sqrt[3]{\frac{(3x+7)^4(x+10)^3}{(5x-8)^2}}$

147) Condense into a single logarithmic expression using the properties of logarithms: $17 \ln x - \frac{2}{3} \ln(x^5 + 5)$

XXI) *Using the graphing calculator:*

148) Graph $y = 0.1x^3 + 2x^2 - x - 3$ on the x - y plane on the right:



149) Find the roots of the equation above.

150) Find the point of intersection for the graphs $y = x^3 + x - 3$ and $y = 2x + 4$.

151) Find the maximum value for the graph $f(x) = -x^4 + x - 4$.

152) For the function in #151, find the intervals on which $f(x)$ is increasing.

XXII) *What are the following trigonometric identities?*

153) $\sec x =$

154) $\csc x =$

155) $\tan x =$

156) $\cot x =$

157) $\cos^2 x - 1 =$

158) $\sec^2 x - 1 =$

159) $\cot^2 x + 1 =$

XXIII) Evaluate the following expressions:

160) $\sin\left(\frac{\pi}{6}\right)$

161) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

162) $\tan\left(\frac{7\pi}{6}\right)$

163) $\cos(0)$

164) $\cos\left(\frac{\pi}{4}\right)$

165) $\csc\left(\frac{-5\pi}{6}\right)$

166) $\sec(\pi)$

167) $\cot\left(\frac{-\pi}{2}\right)$

168) $\sin^{-1}\left(\frac{1}{2}\right)$

169) $\tan\left(\frac{\pi}{2}\right)$

170) $\sin^2\left(\frac{5\pi}{6}\right)$

171) $\cot\left(\frac{2\pi}{3}\right)$

172) $\sin\left(\frac{\pi}{2}\right)$

173) $\cot^{-1}(-1)$

174) $\sec\left(\frac{3\pi}{4}\right)$

175) $\tan^{-1}(-1)$

176) $\csc(\pi)$

177) $\sec^2\left(\frac{\pi}{4}\right)$

XXIV) Sketch one period of the following trigonometric graphs:

178) $y = \sin x$

179) $y = \cos x$

180) $y = \tan x$

181) $y = \sec x$

182) $y = \csc x$

183) $y = \cot x$

XXV) Solve the following trigonometric equations for the given domain:

184) $\sin x = \cos x$ on $[0, 2\pi]$

Answers:

1	$\frac{y+x}{xy}$
2	$\frac{x+1}{x^2}$
3	$1+x$
4	$\frac{1}{x+y}$
5	$\frac{2x+h}{x^2(x+h)}$
6	$(x+4)(x-4)$
7	$(x-3)(x+2)$
8	$(2x+1)(3x-2)$
9	$x(x-5)(4x+1)$

10	Prime (using \mathbb{R} only)
11	$(x^2-15)(x^2+2)$
12	$(x+3)(x^2-3x+9)$
13	$(x-2)(x^2+2x+4)$
14	$2(x^2-3)(2x-3)^2$
16	$x = -8, 3$
17	$x = \pm\sqrt{14}$
18	$x = -\frac{1}{3}, 2$
19	$x = 0, 4$
20	$x = 1, 5, 6$
21	$x = 1600$
22	$x = \frac{z}{y-z}$

23	$x = 26$
24	$x = 3, -3$
25	$x = 9$ (-1 is extraneous)
26	$x = 2$
27	$-1/3$
28	27
29	$[-2, 4]$
30	$\frac{1}{2}$
31	$2, \pm 5$
32	Yes
33	No
34	Yes
35	Yes
36	No

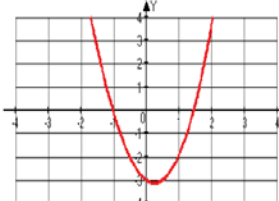
37	$y = 5x - 19$ or $y + 4 = 5(x - 3)$
38	$y - 1 = 3(x - 4)$ or $y + 2 = 3(x - 3)$ or $y = 3x - 11$
39	$y - 1 = -2(x + 2)$ or $y = -2x - 3$
40	$y = 1$
41	$y = \frac{4}{3}x$
42	$x = 4$
43	x-int: (3, 0), (-3, 0); y-int: (0, 3), (0, -3)
44	$y - 0 = -\frac{3}{5}(x - 5)$ or $y - 3 = -\frac{3}{5}(x - 0)$ or $y = -\frac{3}{5}x + 3$
45	$y = \frac{1}{2}x + 1$
46	$y = -\frac{3}{2}x + 3$
47	$x = 4$
48	$y = 1$
49	$x^2 + y^2 = 36$
50-53	Answers vary. Check your rise/run!
54-73	Check using a graphing calc
74	$f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$
75	$f(x) = \begin{cases} x - 1, & x \geq 1 \\ -x + 1, & x < 1 \end{cases}$
76	$D: (-\infty, \infty); R: (-\infty, \infty)$
77	$D: x \neq 0; R: y \neq 0$
78	$D: (-\infty, \infty); R: 0 \leq y \leq 1$ (look at a graph on your calculator to see this)
79	$D: (-\infty, \infty); R: y \geq 0$

80	$D: x \geq 4; R: y \geq 0$
81	$D: (-\infty, \infty); R: y \geq 2$
82	$D: x \geq 0; R: (-\infty, \infty)$
83	$D: x \leq -1$ or $x \geq 4; R: y \geq 0$
84	$D: x \neq -6; R: y \neq -10$
85	$f^{-1}(x) = x - 3$
86	$f^{-1}(x) = x^2$
87	$f^{-1}(x) = \frac{-2x}{x-1}$
88	$f^{-1}(x) = e^x$
89	9
90	$(x+3)^3 + 1$
91	126
92	$x^4 - 4x^2 + 2$
93	$x^3 + 3x^2h + 3xh^2 + h^3 + 1$
94	$3x^2 + 3xh + h^2, h \neq 0$
95	a) (1, 2) b) (-10, 89), (1, 12)
96	a) $P=6w$ b) $w=5 \text{ ft}, l=10 \text{ ft}$
97	(1, 2)
98	(-2, 5), (1, 8)
99	The graph is in the upper right quadrant; all coordinate values are positive
100	The point (2, 5)
101	The graph passes the vertical line test; for every x, there is exactly one y.
102	x-intercept of 4; (4, 0) is on the graph

103	$y=kx$ for some constant k
104	Answers may vary. Example: $x^2 + 4x^3 + 5x + 3$
105	One x-intercept (roots are zeros or x-intercepts of a function)
106	Smooth curve; equation has real number coefficients and positive integer exponents
107	Corresponding angles are congruent, corresponding sides are proportional
108	$f(-x) = f(x)$ Symmetry over the y-axis
109	$f(-x) = -f(x)$ Symmetry over the origin (180 degree rotational symmetry about the origin)
110	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

111	$a^2 + b^2 = c^2$
112	$x\sqrt{2}$
113	$2x$
114	$V = \frac{4}{3}\pi r^3$
115	$V = \pi r^2 h$
116	$V = \frac{1}{3}\pi r^2 h$
117	$V = x^2 h$
118	$A = 4\pi r^2$
119	$A = \pi r^2 + 2\pi r h$
120	$A = \frac{1}{2}bh$
121	$A = \frac{1}{2}h(b_1 + b_2)$
122	$A = \pi r^2$
123	$V = \frac{\sqrt{3}}{4}x^2 y$ (it's okay that you have never seen this before!)
124	$A = \frac{s^2 \sqrt{3}}{4}$ (same with this!)

125	30/7 ft
126	8/3 in
127	$x = 1, y = 0$
128	$x = 1, y = 1$
129	True
130	True
130	False! (There are two #130s...oops!)
131	False!
132	True
133	False!
134	True
135	False!
136	True
137	False!
138	True
139	False!
140	False!
141	False!
142	True
143	True
144	True
145	False!

146	$\frac{1}{3}[4\ln(3x+7) + 3\ln(x+10) - 2\ln(5x-8)]$	148	
147	$\ln\left(\frac{x^{17}}{\sqrt[3]{(x^5+5)^2}}\right)$		

149	1.4389, -1.0211, and -20.4178 (adjust your window to see the entire cubic function!)
150	(2.087, 8.173)
151	-3.528
152	$x < 0.630$
153	$\frac{1}{\cos x}$
154	$\frac{1}{\sin x}$
155	$\frac{\sin x}{\cos x} = \frac{1}{\cot x}$
156	$\frac{\cos x}{\sin x} = \frac{1}{\tan x}$

157	$\sin^2 x$
158	$\tan^2 x$
159	$\csc^2 x$
160	$\frac{1}{2}$
161	$\frac{\pi}{6}$
162	$\frac{\sqrt{3}}{3}$
163	1
164	$\frac{\sqrt{2}}{2}$
165	-2
166	-1
167	0
168	$\frac{\pi}{6}$
169	Undefined

170	$\frac{1}{4}$
171	$-\frac{\sqrt{3}}{3}$
172	1
173	$\frac{3\pi}{4}$
174	$-\sqrt{2}$
175	$-\frac{\pi}{4}$
176	Undefined
177	2
178-183	Check in your calculator
184	$x = \frac{\pi}{4}, \frac{5\pi}{4}$